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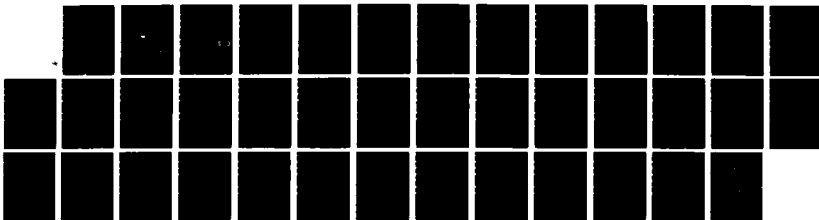
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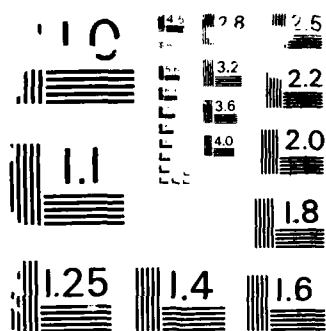
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Interim Technical Report

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TOPOLOGY-SELECTIVE JAMMING OF  
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Interim Technical Report

Andreas Polydoros and Unjeng Cheng

December 23, 1987

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## 1. Introduction

The throughput/delay performance of Code-Division Multiple-Access (CDMA) spread-spectrum networks has received wide attention in the past decade, particularly because of the military interest in mobile packet radio [KaGrBuKu78], [Jubi85], [ShTo85]. Accordingly, the survivability of a Packet Radio (PR) network under jamming attack is an important issue. Conceptually, a network could be attacked on three layers of importance, namely the network, link, and physical layers. In such a jamming game, many factors could affect the results. The communicators have the choices of routing algorithms, channel quality monitoring schemes, and the network information exchange schemes. On the other hand, jammer choices include *temporal* features (such as static, dynamic or follower jamming) as well as *topological* features (i.e., selection of nodes or links to be jammed).

The purpose of the present paper is to introduce certain concepts on *topology-selective jamming* and further examine its impact on a certain class of "local" or "monohop" networks, which we specify below. This particular class of networks has been selected because there exists a convenient analytic vehicle which adequately describes their performance [PoSi87]; thus, it provides a reasonable starting model upon which topological jamming can be defined and assessed with a good degree of analytic ease. We note, however, that the following definitions of topological selectivity are quite general and can be applied to other models as well. We also note that topological selectivity can be perceived as a complementary notion to temporal selectivity, which pertains to jamming patterns with different time-domain profiles but homogeneous with regard to space;<sup>1</sup> the reader is referred to [PrPo87] for a discussion of the latter case.

As mentioned, jamming with any particular selectivity feature manifests itself on all three layers. Thus, it is not immediately clear how the "local" or "monohop" results of this

---

<sup>1</sup>In other words, the jammer can be ON or OFF in a deterministic or stochastic way but, when ON, all nodes are jammed.

paper ought to be interpreted in a larger multihop environment, especially in view of the fact that there does not seem to exist a unique, widely accepted analytic tool for performance evaluation in this case. Here, we choose to focus on the monohop model because (a) certain networks are indeed quite adequately described by this model and (b) it allows for certain conclusions and assessment to be made (one has to crawl before walking).

The paper is organized as follows: In the next Section 2, we first review the basic features of the network model (part 2a); subsequently, we introduce a general probabilistic model for topology-selective jamming (part 2b). For illustrative purposes, we focus upon two particular jamming scenarios which we consider in detail throughout the paper. The corresponding throughput/delay expressions are provided in the following Section 3. An efficient combinational algorithm is also presented for the recursive evaluation of certain important probabilistic parameters, an enhancing feature of the theory in [PoSi87] in its own merit. The paper is concluded with numerical results in Section 4.

## 2. Network and Jamming Models

We identify in this section the particular monohop network and jamming models which we employ in the present study. As mentioned, the monohop network environment under consideration is the one analyzed in [PoSi87], while the probabilistic, topology-selective jamming introduced here is quite general and can be used in conjunction with other network models.

### 2a. Network Model

We consider an arbitrary topology involving  $N_T$  transmitters and  $N_R$  receivers under a symmetry condition, namely, that each transmitter (TR) face the same probabilistic circumstances in the channel; an analogous symmetry holds from each receiver's (RCVR's) viewpoint. This local network is fully-connected, i.e., every TR can be heard from every RCVR. Time is slotted. We shall assume that the number of receivers  $N_R$  is *fixed*, and that all RCVR's are always available (dedicated receivers). In terms of the notation of Table I (repeated from [PoSi87], for convenience), this implies that  $M_R = N_R$ , i.e., all RCVR's are always active. Such a scenario includes the full-duplex case as a special one ( $N_T = N_R = U$ ), but specifically excludes the half-duplex case whereby a modem is an active RCVR whenever it is not transmitting ( $M_R = U - M_T$ ). Note that, in all cases, the number of active TR's  $M_T$  is a random variable ( $0 \leq M_T \leq N_T$ ) with an unconditional pdf  $f_{M_T}(m)$ . Let us note again that certain extensions to, say, the half-duplex model are straightforward but will not be discussed here.

The access protocol is of the slotted ALOHA type with  $p_0$ ,  $p_r$  denoting the new-packet transmission and backlogged-packet retransmission probabilities, respectively [KILa75]. We shall assume a finite number of TR's, although extension to the infinite Poisson model is again straightforward. The effect of buffering is not considered here; thus, users are always busy (never idle with any empty buffer), meaning that they are either active with a new packet or backlogged with an old one (this is the standard model of

TABLE 1  
Key Parameters

$U$	Total number of radio units in the local channel (fixed)
$N_T$	Maximum number of potential transmitters in a slot (fixed)
$N_R$	Maximum number of potential receivers in a slot (fixed)
$M_T$	Number of active transmitters in a particular slot (r.v.)
$M_R$	Number of active receivers in a particular slot (r.v.)
$B$	Number of backlogged users at the beginning of a slot (r.v.)
$M_B$	Number of backlogged users retransmitting in a slot (r.v.)
$M_0$	Number of new users transmitting in a slot (r.v.)
$S$	Number of "channel successes" in a slot (r.v.)
$f_{M_T}(m)$	Prob $\{M_T = m\}$ , unconditional pdf
$\Delta(\xi, \Xi, p)$	$\Pr\{\xi \text{ successes in } \Xi \text{ trials}\}$ , binomial with parameter $p$

[KILa75]). For a network under jamming stress, the assumption of never-idle users is quite reasonable and corresponds to the interesting case of the jammer being sufficiently strong to keep the network busy most of the time.

Finally, regarding the spreading code distribution, we assume either a common code or TR-based code system, thus specifically excluding RCVR-based codes [SoSi84], [SuLi85], [Purs87]. The implication is that a TR's packet can be successfully received by more than one RCVR, although it could potentially contribute only one unit to the total success (or throughput) count.<sup>2</sup> We have termed such as scenario competitive to account for the TR's "effort" to secure some RCVR's attention. This is in contrast to the paired-off scenario [PoSi87], [PrPo87] (which can be thought of as a simultaneous TR and RCVR-based system) where TR's and RCVR's form distinct pairs and only suffer from secondary multi-user interference, while the RCVR attention is not an issue. A comparison between the competitive and paired-off cases should provide an indication of the impact of the user monohop topology upon the jamming game.

## 2b. Jamming Model

We adopt the following probabilistic model for the jamming action: the slotted timing is perfectly known to the jammer, so that perfect slot synchronization exists (the cost of removing such an assumption can be dealt with in a way similar to [PrPo87]). The jammer performs an independent probabilistic action on a slot-by-slot basis. In each slot, a random number of RCVR's,  $M_R^J$ , is selected for jamming out of the  $N_R$  total. The selection of the specific jammed subset is independent from slot-to-slot, so that no memory can be exploited by the users. Note that the jammer knows exactly the location of the dedicated RCVR's. Also, the jamming pattern is assumed independent of the channel state (e.g., the number of backlogged users), a realistic scenario for a rapidly changing environment.

---

<sup>2</sup>Alternatively, one can count as many successes as receptions regardless of origin; this would seem to be a more appropriate performance measure in a flooding-type of routing [PoSi87].

However, extensions to a channel-dependent scheme would also be of interest, possibly necessitating a different probabilistic jamming model than the one herein.

A particular topology-selective jamming strategy manifests itself in the way the  $M_R^J$  jammed RCVR's are selected. If we define the binary-valued r.v.'s  $A_i^J$ ;  $i=1, \dots, N_R$  as

$$A_i^J = \begin{cases} 1, & \text{if RCVR}_i \text{ is jammed} \\ 0, & \text{otherwise} \end{cases} \quad (1)$$

then we can quantify topological selectivity by the joint probability mass distribution function

$$\Pr[\underline{A}^J = \underline{a}] \triangleq \Pr[A_1^J = a_1, \dots, A_{N_R}^J = a_{N_R}] \quad (2)$$

where  $a_i = 0$  or  $1$ ;  $i = 1, \dots, N_R$ . Thus, the experiment performed by the jammer consists of drawing a random vector  $\underline{A}^J$  in each slot, which then identifies the targeted RCVR's. The assumption of independent trials (per slot) plus any given distribution (2) completely determine the underlying probability measure.

Associated with the above are two important parameters, namely (a) what we shall call the *spatial duty factor*,  $\rho_{sp}$ , defined as<sup>3</sup>

$$\rho_{sp} \triangleq \frac{\mathcal{E}\{M_R^J\}}{N_R} \quad (3)$$

signifying the average fraction of jammed RCVR's and (b) the jamming power per attacked RCVR per slot,  $J_{RCVR}$ , assumed fixed throughout, as it relates to the average jamming power  $J_{av}$ :

$$J_{RCVR}(\rho_{sp}) \triangleq \frac{J_{av}}{\mathcal{E}\{M_R^J\}} = \left( \frac{J_{av}}{N_R} \right) \rho_{sp}^{-1} \quad (4)$$

---

<sup>3</sup>Here,  $\mathcal{E}\{\cdot\}$  stands for expectation.

In the following analysis we shall assume that the jammer is average-power-limited, i.e.,  $J_{av}$  is constant and given. Thus, varying the spatial duty factor presents the classical tradeoff between percentage of jammed RCVR's versus the power  $J_{RCVR}(\rho_{sp})$  directed to each one of them. The functional dependence of equation (4) on  $\rho_{sp}$  is meant to enhance this point.

Definition (2) is quite general and can serve as a starting point for different probabilistic jamming options. Here, we focus on two special cases which we shall call scenario 1 and 2, respectively:

**Scenario 1:** Each RCVR is jammed with probability  $p_J$ , independent of any other RCVR. Then,

$$\Pr[\underline{A}^J = \underline{a}] = p_J^{\sum a_i} (1 - p_J)^{N_R - \sum a_i} \quad (5)$$

where  $\sum a_i \triangleq m_R^J(\underline{a})$  is the standard Hamming weight of the binary vector  $\underline{a}$ . Note that  $m_R^J(\underline{a})$  is simply the value of the r.v.  $M_R^J$  in that slot. Consequently,  $\mathcal{E}\{M_R^J\} = p_J N_R$ , which simply implies from (3) that

$$\rho_{sp} \equiv p_J \quad (\text{Scenario 1}) . \quad (6)$$

**Scenario 2:** The number of RCVR's jammed is fixed to  $N_R^J$  (a given constant), although the specific subset changes randomly every slot. Then,

$$\Pr[\underline{A}^J = \underline{a}] = \begin{cases} \left( \frac{N_R^J}{N_R} \right)^{N_R^J} , & \text{if } m_R^J(\underline{a}) = N_R^J \\ 0 , & \text{otherwise} \end{cases}$$

regardless of the exact location of 1's in  $\underline{a}$ . Clearly,

$$\rho_{sp} = \frac{N_R^J}{N_R} \quad (\text{Scenario 2}) . \quad (8)$$

Note that  $\rho_{sp}$  of (8) is restricted to multiples of  $N_R^{-1}$ , while  $\rho_{sp}$  of Scenario 1 can take on any value in  $(0,1]$ .



### 3. Performance Analysis

A network under the jamming model of the previous section can be analyzed by generalizing the concepts in [PoSi87]. In particular, we shall assume that the probabilistic symmetry conditions of the above model are still true and that the probability distribution of the number of successes per slot can be uniquely determined, once the number of attempted transmissions  $M_T = m$  and the specific jamming pattern  $\underline{A}^J = \underline{a}$  are given for that slot. We can evaluate the throughput  $\beta$  (packets/slot) as the expected number of successful packet transmissions  $S$ , i.e.,

$$\begin{aligned}\beta &= \mathcal{E}\{S\} = \mathcal{E}_{M_T, \underline{A}^J} \left\{ \mathcal{E}\{S | M_T, \underline{A}^J\} \right\} \\ &= \sum_{\underline{a}} \Pr[\underline{A}^J = \underline{a}] \mathcal{E}_{M_T} \left\{ \mathcal{E}\{S | M_T, \underline{a}\} \right\} \\ &= \sum_{\underline{a}} \Pr[\underline{A}^J = \underline{a}] \sum_m m p_s^T(m, \underline{a}) f_{M_T}(m)\end{aligned}\quad (9)$$

where  $f_{M_T}(m)$  is the composite slot traffic (Table I) and  $p_s^T(m, \underline{a})$  is the probability of success from a typical TR's viewpoint, given another  $(m - 1)$  packets and a specific jamming pattern  $\underline{a}$  in the slot. In deriving (9), we have used the fact that the r.v.  $M_T$  (attempted transmissions) is independent of  $\underline{A}^J$ , since there is no coupling between the users' and jammer's actions within a slot. Upon interchanging the summations in (9) and defining the jamming-average probability of success from a TR's viewpoint, conditioned on another  $(m - 1)$  packets, as

$$\overline{p_s^T(m)} \triangleq \sum_{\underline{a}} \Pr[\underline{A}^J = \underline{a}] p_s^T(m, \underline{a}) \quad (10)$$

we arrive at

$$\beta = \sum_m m \overline{p_s^T(m)} f_{M_T}(m) \quad (11)$$

which is a direct generalization of (42) in [PoSi87]. In order to proceed, we need to

evaluate  $\overline{p_s^T(m)}$  and  $f_{M_T}(m)$  for any specific jamming strategy as per (2).

### 3a Evaluation of $\overline{p_s^T(m)}$

In principle,  $\overline{p_s^T(m)}$  can be evaluated in accordance with *Proposition 1* of [PoSi87]

(which is quite general and also holds in this jammed scenario) as

$$\overline{p_s^T(m)} = \frac{1}{m} \sum_{s=1}^m \overline{p_{s|m}} \quad (12a)$$

where

$$\overline{p_{s|m}} \triangleq \sum_{\underline{a}} \Pr[\underline{A}^J = \underline{a}] p_{s|m,\underline{a}} \quad (12b)$$

is the average (over the jamming strategy) of the probability

$$p_{s|m,\underline{A}^J} = \Pr[s \text{ successes, given } m \text{ attempted transmissions} \\ \text{and jamming pattern } \underline{A}^J] \quad (13)$$

For the nonjammed scenario, [PoSi87] provides examples of how to calculate  $p_{s|m}$  in a variety of situations in terms of the number of RCVR's, their statistical dependence, etc. This can be a very complicated procedure for arbitrary models; however, a simplification occurs if one assumes that, conditioned on a specific jamming pattern in a slot, each RCVR accepts packets in a statistically independent fashion from other RCVR's. This assumption, which would be obviously valid if receiver thermal-noise were the only deterrent, can be argued to be numerically satisfactory even in the presence of multi-user noise because of random spreading patterns, fading, random distances, ground propagation and formation, etc. Then,  $\overline{p_{s|m}}$  can be evaluated in a recursive way which we shall present in the following section, since it is also required in the evaluation of  $f_{M_T}(m)$ . Let us just note here that, once this evaluation is completed,  $\overline{p_s^T(m)}$  follows from (12).

Under the independence assumption outlined above, a shorter path for evaluating

$\overline{p_s^T(m)}$  is as follows: Let  $p_A^R(m;J)$  and  $p_A^R(m;J^c)$  denote the probabilities that a typical RCVR will accept a packet in the presence of another  $(m-1)$  contending ones, given that this RCVR is jammed (event  $J$ ) or not (event  $J^c$ ), respectively:<sup>4</sup> Then, using (10) and an immediate generalization of *Proposition 2* of [PoSi87], we get

$$\begin{aligned} \overline{p_s^T(m)} &= \sum_{\underline{a}} \Pr[\underline{A}^J = \underline{a}] \left[ 1 - \left( 1 - \frac{p_A^R(m;J^c)}{m} \right)^{N_R - m_R^J(\underline{a})} \left( 1 - \frac{p_A^R(m;J)}{m} \right)^{m_R^J(\underline{a})} \right] \\ &= \sum_{m_R^J=0}^{N_R} \Pr(M_R^J = m_R^J) \left[ 1 - \left( 1 - \frac{p_A^R(m;J^c)}{m} \right)^{N_R - m_R^J} \left( 1 - \frac{p_A^R(m;J)}{m} \right)^{m_R^J} \right] \end{aligned} \quad (14)$$

where  $M_R^J$  is the Hamming-weight transformation (a r.v.) of the random vector  $\underline{A}^J$ . Note the impact of the aforementioned symmetry assumption, which implies here that all jammed RCVR's suffer the same interference level.

To illustrate, consider again the two scenarios of section 2b. We immediately have that

$$\Pr(M_R^J = m_R^J) = \Delta(m_R^J, N_R, \rho_{sp}) \quad ; \quad \text{scenario 1} \quad (15a)$$

and

$$\Pr(M_R^J = m_R^J) = \begin{cases} 1, & \text{if } m_R^J = N_R^J \\ 0, & \text{otherwise} \end{cases} \quad ; \quad \text{scenario 2} \quad (15b)$$

which, when substituted in (14), yield

---

<sup>4</sup>Note that  $p_A^R(m;J^c)$  is identical to  $p_A^R(m)$ , as introduced in [PoSi87]. Here, because of the possible jamming, we need to distinguish further and define these conditional quantities.

$$\overline{p_s^T(m)} = \begin{cases} 1 - \left\{ 1 - \frac{\rho_{sp} p_A^R(m;J) + (1 - \rho_{sp}) p_A^R(m;J^c)}{m} \right\}^{N_R} & \text{(scenario 1)} \\ 1 - \left\{ \left( 1 - \frac{p_A^R(m;J)}{m} \right)^{\rho_{sp}} \left( 1 - \frac{p_A^R(m;J^c)}{m} \right)^{1-\rho_{sp}} \right\}^{N_R} & \text{(scenario 2)} . \end{cases} \quad (16)$$

It should be kept in mind that  $p_A^R(m;J)$  above depends on  $\rho_{sp}$  via the jamming level  $J_{RCVR}(\rho_{sp})$ . Note also that the result for scenario 1 depends only on the *jamming-average acceptance probability*

$$\overline{p_A^R(m)} \triangleq \rho_{sp} p_A^R(m;J) + (1 - \rho_{sp}) p_A^R(m;J^c) \quad (17)$$

which is an intuitively expected result, because RCVR's are jammed independently for this case. Obviously, this is not true for scenario 2.

For the specific protocol choice  $p_0 = p_r = p$  (the "backlog-independent protocol" of [PrPo87]) the composite traffic  $f_{M_T}(m)$  is simply the binomial distribution  $\Delta(m, N_T, p)$ , regardless of any jamming action. For this limited case, equations (11) and (16) suffice to determine throughput. The more general and interesting case, however, is when  $p_0 \neq p_r$ , which we examine below.

### 3b. Evaluation of $f_{M_T}(m)$ and $\overline{p_{s|m}}$

Following closely the steps in [PoSi87, Appendix A], we can evaluate  $f_{M_T}(m)$  as

$$f_{M_T}(m) = \sum_{b=0}^{N_T} \pi_B(b) f_{M_T|B=b}(m|b) \quad (18)$$

where  $f_{M_T|B=b}(m|b)$ , the conditional composite traffic given  $b$  backlogged users at the beginning of the slot, is evaluated from

$$f_{M_T|B=b}(m|b) = \sum_n \Delta(m-n, b, p_r) \Delta(n, N_T-b, p_0) \quad (19)$$

$$\max(0, m-b) \leq n \leq \min(N_T-b, m)$$

with  $(p_0, p_r)$  the first transmission and retransmission probabilities, respectively. Note that (19) is independent of the jamming action. In (18),  $\pi_B(b)$  is the appropriate eigenvector of the jamming-average matrix  $\overline{P} = \{ \overline{p_{ij}} \}$ , i.e.,

$$\pi_B(b) = \pi_B(b) \overline{P} \quad (20)$$

where<sup>5</sup>

$$\overline{p_{ij}} = \sum_{m_0=\max(0, j-i)}^{N_T-i} \Delta(m_0, N_T-i, p_0) \cdot \sum_{m_b=\max(0, j-i)}^i \Delta(m_b, i, p_r) \overline{p_{i+m_0-j|m_0+m_b}} \quad (21)$$

It is clear from (18), (20), (21) that knowledge of the set  $\{ \overline{p_{s|m}} \}$  is fundamental in the evaluation of the composite traffic. Assuming, as in (13), that all patterns  $\underline{a}$  with the same Hamming weight  $m_R^J(\underline{a})$  result in the same  $p_{s|m, \underline{a}}$ , we can rewrite (12b) as

<sup>5</sup>Note that the eigenvector  $\pi_B(b)$  of the composite system is not itself a jamming-average of eigenvectors, hence we should not write  $\overline{\pi_B(b)}$ .

$$\overline{p}_{sim} = \sum_{m_R^J} \Pr[M_R^J = m_R^J] p_{sim, m_R^J} \quad (22)$$

In order to evaluate  $p_{sim, m_R^J}$ , we shall assume that the  $M_R$  receivers operate independently, conditioned on the jamming pattern. Let  $\mathcal{R}^J \triangleq \{\text{RCVR}_k, k=1, \dots, m_R^J\}$  and  $\mathcal{R}^{J^c} \triangleq \{\text{RCVR}_k, k=m_R^J+1, \dots, N_R\}$  indicate that jammed and nonjammed sets of receivers, respectively. Let  $p_A^{R_k}$  generally stand for the probability of acceptance from the  $k^{\text{th}}$  RCVR's viewpoint. Under the symmetry assumption for each of the sets  $\mathcal{R}^J, \mathcal{R}^{J^c}$ , we then have that

$$p_A^{R_k} = \begin{cases} p_A^R(m; J), & \text{if } \text{RCVR}_k \in \mathcal{R}^J (k \leq m_R^J) \\ p_A^R(m; J^c), & \text{if } \text{RCVR}_k \in \mathcal{R}^{J^c} (k > m_R^J) \end{cases} \quad (23)$$

Note that, in the present probabilistic framework, which individual RCVRs belong to the sets is irrelevant, and their ordering can be arbitrary; it is the set size  $m_R^J = \dim \mathcal{R}^J$  that counts.

A recursive way to calculate  $p_{sim, m_R^J}$  is the following: let  $p_{sim, m_R^J}^{(k)}$  denote the above probability of  $s$  successes, given  $m$  attempts and  $m_R^J$  jammed receivers, which is due to the first  $k$  receivers,  $k = 0, \dots, N_R$ . Clearly, the sought probability is simply  $p_{sim, m_R^J} = p_{sim, m_R^J}^{(N_R)}$ . We can first define

$$p_{sim, m_R^J}^{(0)} = \begin{cases} 1 & , \quad s = 0 \\ 0 & , \quad \text{otherwise} \end{cases} \quad (24a)$$

Now consider the first receiver  $\text{RCVR}_1$ . Then

$$p_{slm, m_R^J}^{(1)} = \begin{cases} 1 - p_A^{R_1} & , s = 0, m \geq 1 \\ p_A^{R_1} & , s = 1, m \geq 1 \\ 0 & , s \geq 2 \text{ or } m = 0 \end{cases} \quad (24b)$$

where  $p_A^{R_1}$  has been defined in (23) as a function of  $m$  and  $m_R^J$ . Generalizing to the  $k^{\text{th}}$  RCVR's, we can use a standard combinatorial method to conclude that

$$p_{slm, m_R^J}^{(k)} = \begin{cases} (1 - p_A^{R_k}) p_{0lm, m_R^J}^{(k-1)} & , s = 0 \\ \left[ (1 - p_A^{R_k}) + \frac{s}{m} p_A^{R_k} \right] p_{slm, m_R^J}^{(k-1)} + \\ + \frac{m - s + 1}{m} p_A^{R_k} p_{s-1lm, m_R^J}^{(k-1)} & , 1 \leq s \leq \min(m, k) \\ 0 & , s > k \text{ or } m \end{cases} \quad (24c)$$

The recursion stops when  $k = N_R$ . Note that two different quantities will be used in the place of  $p_A^{R_k}$  above, depending on whether  $k \leq m_R^J$  or  $k > m_R^J$ , as per (23).

The above recursion can be used with any specific jamming number  $m_R^J$  to produce  $p_{slm, m_R^J}$  and then, via (22),  $\overline{p_{slm}}$ . It is also useful for calculating  $p_{slm}$  in a nonjammed environment (thermal and multi-user noise only), thus augmenting the theory of [PoSi87].

Regarding our previous two scenarios, we note that

$$\overline{p_{slm}} = p_{slm, N_R^J}^{(N_k)} ; \quad \text{scenario 2} \quad (25)$$

which is a direct result of (15b) and (24). For scenario 1, it can be shown (see Appendix

A) that  $\overline{p_{slm}}$  can be obtained directly from the recursion (24) (i.e., substitute  $p_{slm, m_R^J}^{(k)}$  for

$p_{slm}^{(k)}$  and identify  $p_{slm, m_R}^{(NR)} \equiv \overline{p_{slm}}$ , so long as we use the jamming average  $\overline{p_A^R(m)}$  of (17) instead of  $p_A^{Rk}$ . This is also an intuitively appealing conclusion, in view of the fact that all RCVRs are mutually independent and statistically identical under scenario 1, each described probabilistically by  $\overline{p_A^R(m)}$ .

It is instructive to compare the above analysis for topology-selective jamming with the dual concept of *temporal selectivity*. We take this issue up in the following section.

### 3c Comparison with Temporal Jamming

In our terminology, temporal implies a pulsed (blinking) two-level (ON-OFF) jamming pattern which, when ON, covers all local receivers under consideration with the same power  $J_{RCVR}$ . Let  $\rho_t$  indicates the temporal duty-factor of the jammer. It represents the long-term fraction of time that the jammer is ON, as well as the probability that a randomly observed slot will be found in the jamming state. As explained in [PrPo87], a variety of jamming waveforms can be constructed which have the same  $\rho_t$ , but different sample paths. In essence, one can vary the length (in slots) of the ON or OFF sessions by different probabilistic mechanisms, yet keep  $\rho_t$  fixed.

The two extreme cases, from a temporal variability or slot-correlation viewpoint, are (a) a long-term jammer, which stays in the same ON or OFF state for very long (practically, infinite intervals of time and (b) a slot-by-slot independent jammer with jamming probability  $\rho_t$ . The mathematical model for (a) is that of initially chasing between a "good" channel and a "bad" channel with probability  $\rho_t$  and  $(1 - \rho_t)$ , respectively, and staying there forever. In all cases, we can express total throughput as

$$\beta = \rho_t \beta_J + (1 - \rho_t) \beta_{jc} \quad (26)$$

where the different temporal jamming strategies manifest themselves in the way we evaluate



the conditional throughputs  $\beta_J$  and  $\beta_{J^c}$ . For the long-term jammer, it is immediate that

$$\beta_J = \sum_m m p_s^T(m; \mathcal{J}) f_{M_T}(m | \mathcal{J}); \quad \mathcal{J} = J \text{ or } J^c \quad (27)$$

where the symbol  $\mathcal{J}$  is used as a jamming index; thus, (26) is easily evaluated, virtually by analyzing a multiple-access channel in two different interference levels.

It can be argued that (27) holds for all cases exactly as (26) does, except that  $f_{M_T}(m | \mathcal{J})$  should be with finite jamming block size, interpreted as the conditional stationary distribution of transmitting users, evaluated for the state- $\mathcal{J}$  slots only ( $\mathcal{J} = J$  or  $J^c$ ). In other words,  $f_{M_T}(m | \mathcal{J})$  represents the probability of having  $M_T = m$  transmissions in a "typical" or randomly-chosen slot, assuming we only look at those slots of status  $\mathcal{J}$ . Unfortunately, evaluation of these two conditional stationary distributions is not trivial, necessitating the solution of a composite Markov chain of size which grows very quickly with complexity; the interested reader is referred to [PrPo87]. There is, however, one case which is significantly simpler, namely the slot-by-slot independent jamming of case (b) above. Then, a little thought will reveal that

$$f_{M_T}(m | J) = f_{M_T}(m | J^c) = f_{M_T}(m) \quad (28)$$

meaning that any slot is a "typical" slot, regardless of whether it belongs to a jammed or nonjammed block. This is precisely so by virtue of the memoryless property of the jamming mechanism from one slot to the next. Substituting property (28) into (26), (27) (recall that the latter two equations hold for any scenario). We arrive at

$$\beta = \sum_m m \overline{p_s^T(m)} f_{M_T}(m) \quad (29a)$$

where

$$\overline{p_s^T(m)} \stackrel{\Delta}{=} \rho_t p_s^T(m;J) + (1 - \rho_t) p_s^T(m;J^c) \quad (29b)$$

is the temporal jamming-average probability of success.

Equation (29) is formally identical to (10), (11), the apparent difference being that averaging here is performed over the temporal profile of the jamming process, as opposed to the spatial jamming profile of Section 3. The root of the similarity is, of course, the fact that the jamming decisions are independent from slot to slot. Furthermore, it is not hard to see that this special case of memoryless temporal jamming can be reformulated in the topology-selective framework by letting

$$\Pr [\underline{A}^J = \underline{a}] = \begin{cases} \rho_t & , \text{ if } a_i = 1 \text{ for all } i \\ 1 - \rho_t & , \text{ if } a_i = 0 \text{ for all } i \end{cases} \quad (30)$$

since a substitution of (30) into (10) will immediately yield (29b)

The last step for case (b) regards the evaluation of  $f_{M_T}(m)$  in (28a). If we incorporate the above conceptual linkage, manifested by (30), into the procedure outlined in (3b) for spatial jamming, we immediately conclude that  $f_{M_T}(m)$  can be obtained from (18) – (21), with the temporal average

$$\overline{p_{slm}} = \rho_t p_{slm,J} + (1 - \rho_t) p_{slm,J^c} \quad (31)$$

resulting from substituting (30) into (22). This is quite a convenient simplification which, as we mentioned, does not occur for jamming patterns with slot memory.

#### 4.0 Numerical Results

The main interest of this investigation is the network aspect of the jamming threat. Thus in the subsequent discussion, we assume the simplest signal format and jamming format, namely, the continuous tone jamming and the coded direct-sequence binary-phase-shift-key modulation. The probabilities of successful reception  $p_A^R(m, J)$  and  $p_A^R(m, J^c)$  are given by

$$p_A^R(m, \Lambda) = \sum_{i=0}^e \binom{L}{i} p_{CE}^i(m, \Lambda) (1 - p_{CE}(m, \Lambda))^{L-i} \quad (32)$$

for the  $e$ -error correction code of block length  $L$  (it is also the packet length here). We also let  $\Lambda = J$  if the receiver is jammed and let  $\Lambda = J^c$  if the receiver is not jammed.  $p_{CE}(m, \Lambda)$  is the channel symbol error rate, which is given by

$$p_{CE}(m, \Lambda) = Q \left[ \sqrt{\frac{2E_{cs}}{N_{eq}(m, \Lambda)}} \right] \quad (33)$$

where

$$\begin{aligned} \frac{2E_{cs}}{N_{eq}(m, J)} &= \frac{G_{baud}}{\gamma_{in}^{-1} + (m-1)\alpha_{ma} + \frac{J}{S}} \\ \frac{2E_{cs}}{N_{eq}(m, J^c)} &= \frac{G_{baud}}{\gamma_{in}^{-1} + (m-1)\alpha_{ma}} \end{aligned} \quad (34)$$

and

$$Q(x) = \frac{1}{\sqrt{2\pi}} \int_x^{\infty} e^{-x^2/2} dx \quad (35)$$

In the above equations,  $G_{baud}$  is the spreading ratio of the channel symbols,  $\gamma_{in}$  is related to the signal-energy-to-thermal noise ratio per channel symbol by  $\gamma_{in} = E_{cs}/(N_0 G_{baud})$ , and

$\alpha_{ma}$  is the multiple-access coefficient depending on the cross-correlation between the particular multiple-access codes in use. The numerical results are computed for  $\alpha_{ma} = 1$  and  $L = 1024$ . For the extended BCH code of block length 1024, the number of correctable errors  $e$  and the coding rate can be related by

$$r = 1 - \frac{11e}{1024} \quad (36)$$

approximately.

In Figures 1 through 3, we show the normalized throughput versus the spatial duty factor  $\rho_{sp}$  with  $p_0$  and  $p_r$  being the parameters for jamming scenario 2. The ten-error correction code of rate 0.89 is used. We see that if  $G_{baud} = 13$  dB, the worst-case  $\rho_{sp}$  approaches to 1 as  $p_0$  and  $p_r$  approach 0.5, and if  $G_{baud} = 20$  dB, worst-case  $\rho_{sp}$  stays about 0.5 for  $p_0$  and  $p_r$  less than 0.5. This phenomena can be explained as follows: Note that the multiple-access self-interference noise is small if  $p_0$  and  $p_r$  are small for a small  $G_{baud}$ , or if  $G_{baud}$  is large. If the self-interference noise is small, the worst-case  $\rho_{sp}$  is less than 1 because the strong jamming power is needed to wipe out each jammed receiver. On the other hand, if the self-interference noise is large, the worst-case  $\rho_{sp}$  is equal to 1 because the small jamming power is sufficient to wipe out each jammed receiver.

In Figures 4 through 6, we show the normalized throughput versus the spatial duty factor  $\rho_{sp}$  with  $p_0$  being the parameters for jamming scenario 2. The retransmission probability  $p_r$  is optimized in these three figures. We see that the effectiveness of the worst-case  $\rho_{sp}$  is slightly reduced for many cases by optimizing  $p_r$ .

In Figure 7, we show the normalized throughput versus the jammer spatial duty factor  $\rho_{sp}$  with  $p_0 = p_r = 0.3$  for several error-correction codes for jamming scenario 2. In the same figure, we also show the normalized throughput for the paired-off case. Notice that the competitive scenario is more robust against the worst-case jamming than the paired-off scenario.

Figure 1. Normalized throughput versus the spatial duty factor  $\rho_{sp}$  for jamming scenario 2.

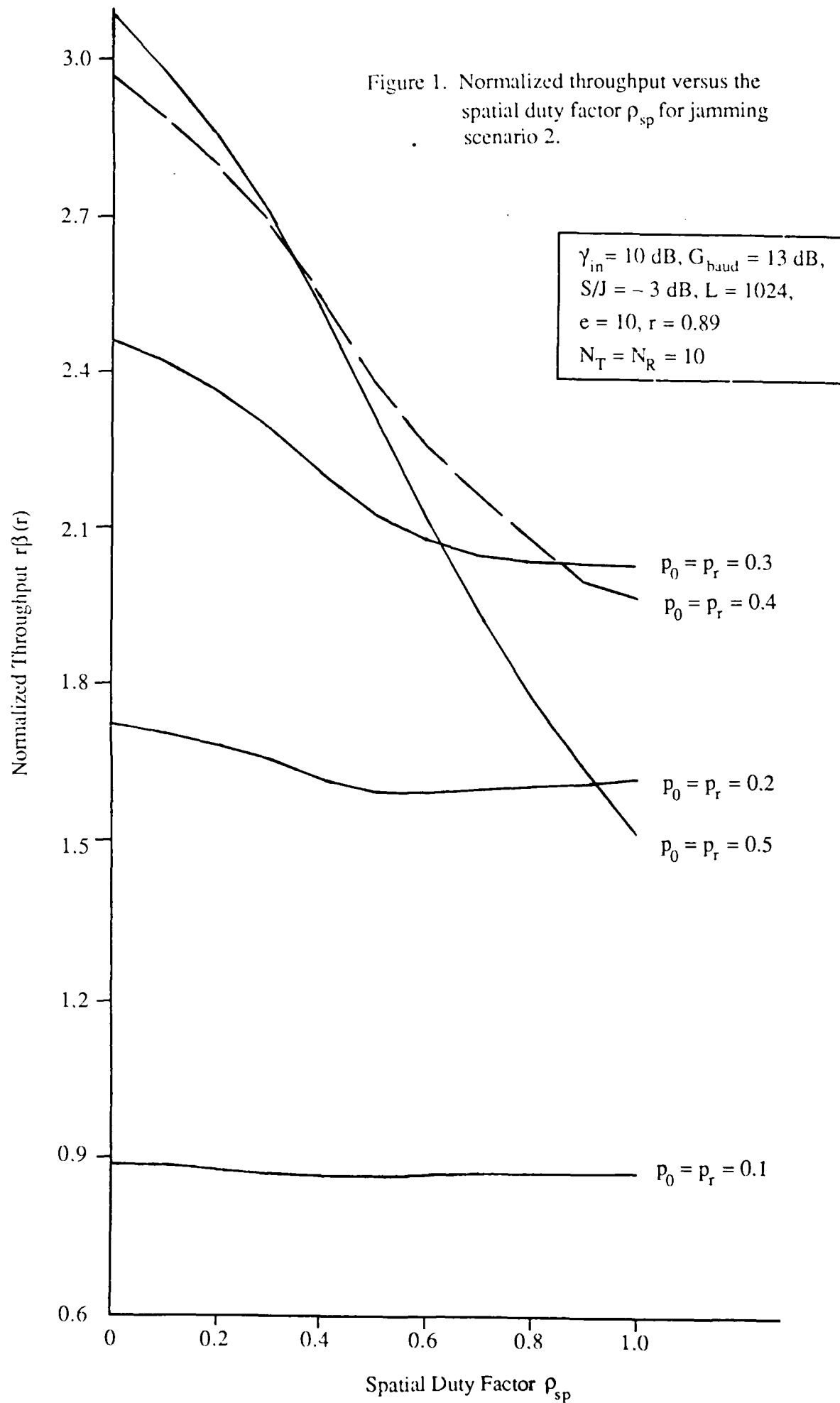


Figure 2. Normalized throughput versus the spatial duty factor  $\rho_{sp}$  for jamming scenario 2.

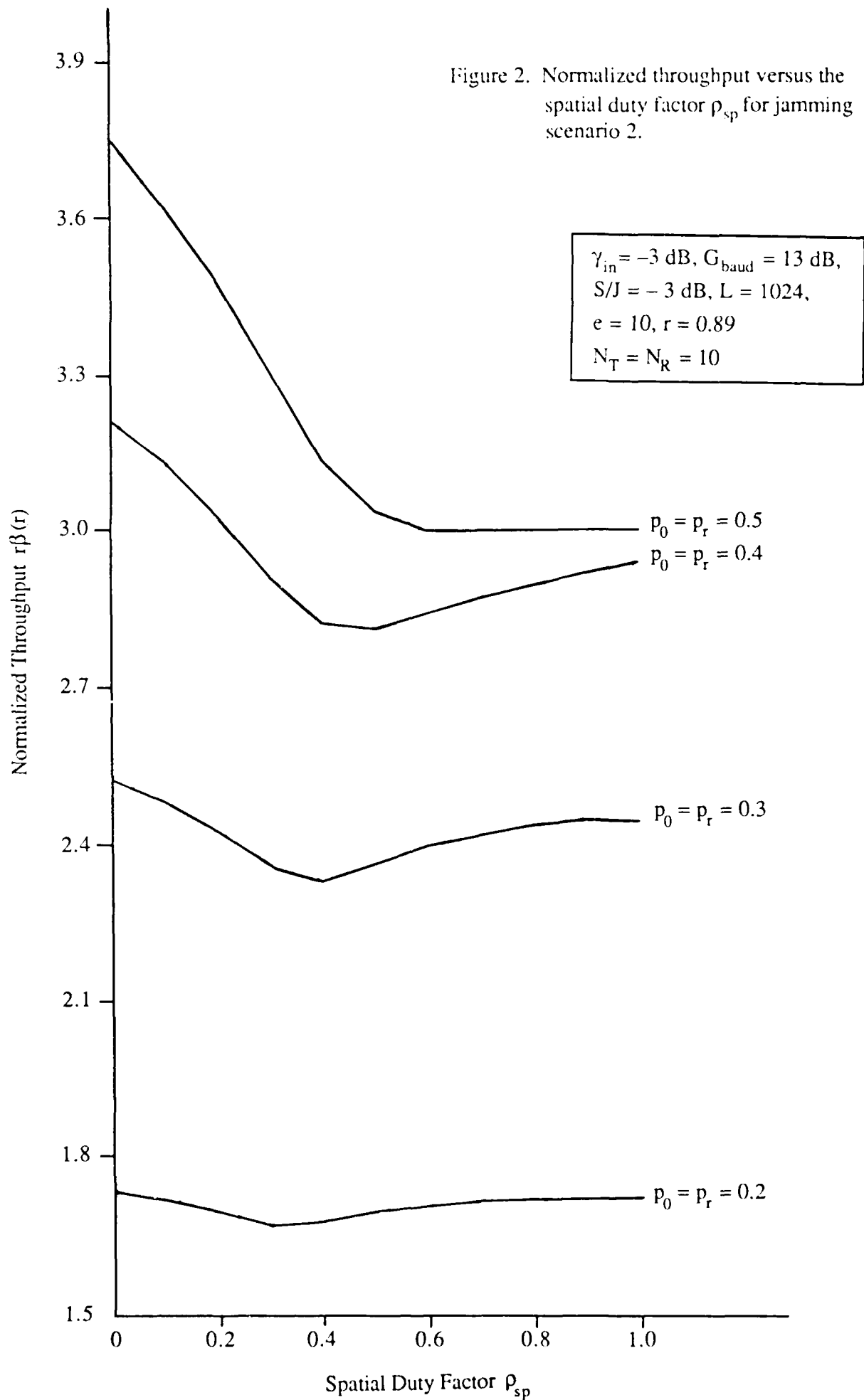


Figure 3. Normalized throughput versus the spatial duty factor  $\rho_{sp}$  for jamming scenario 2.

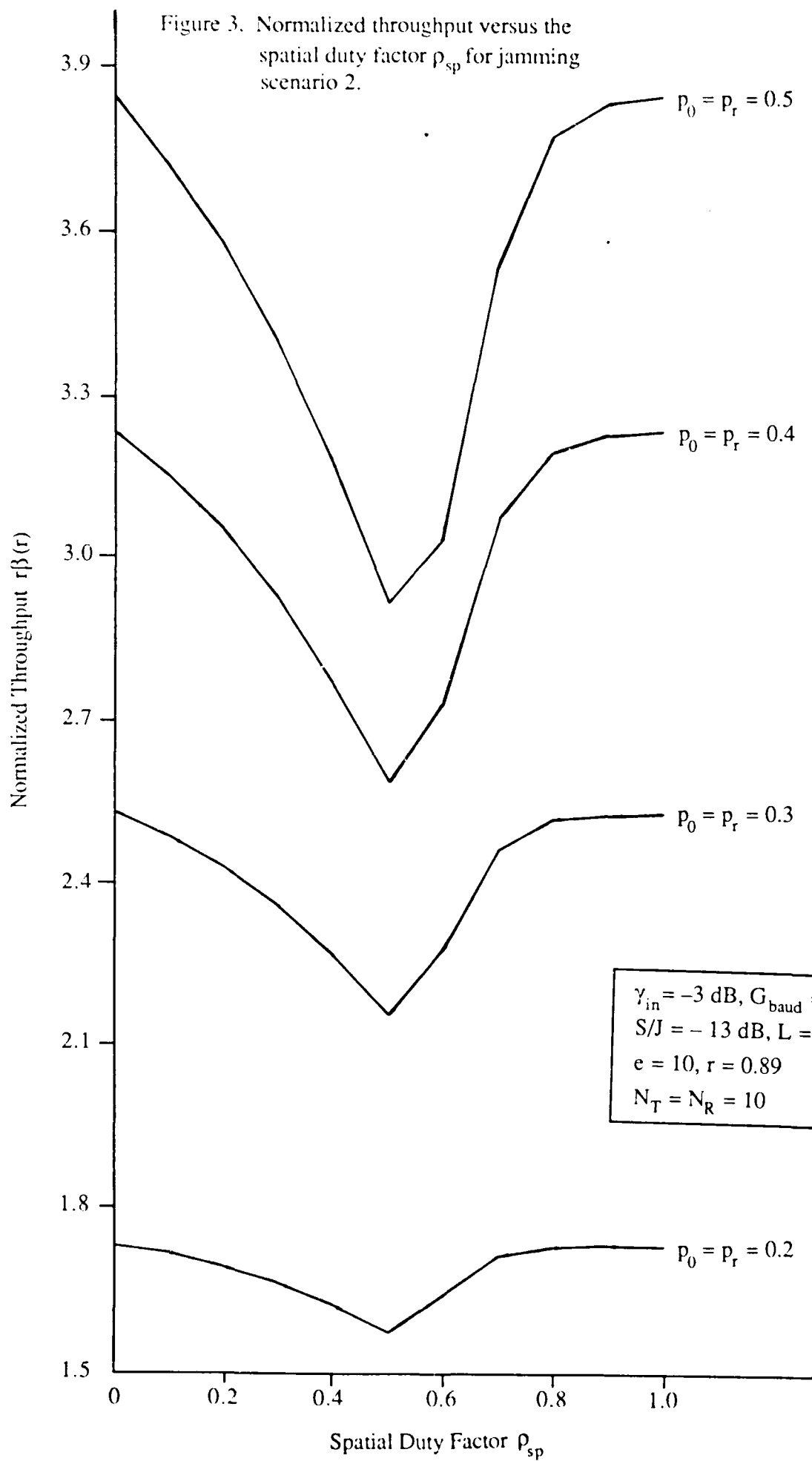


Figure 4. Normalized throughput versus the spatial duty factor  $\rho_{sp}$  for jamming scenario 2. The retransmission probability  $p_r$  is optimized.

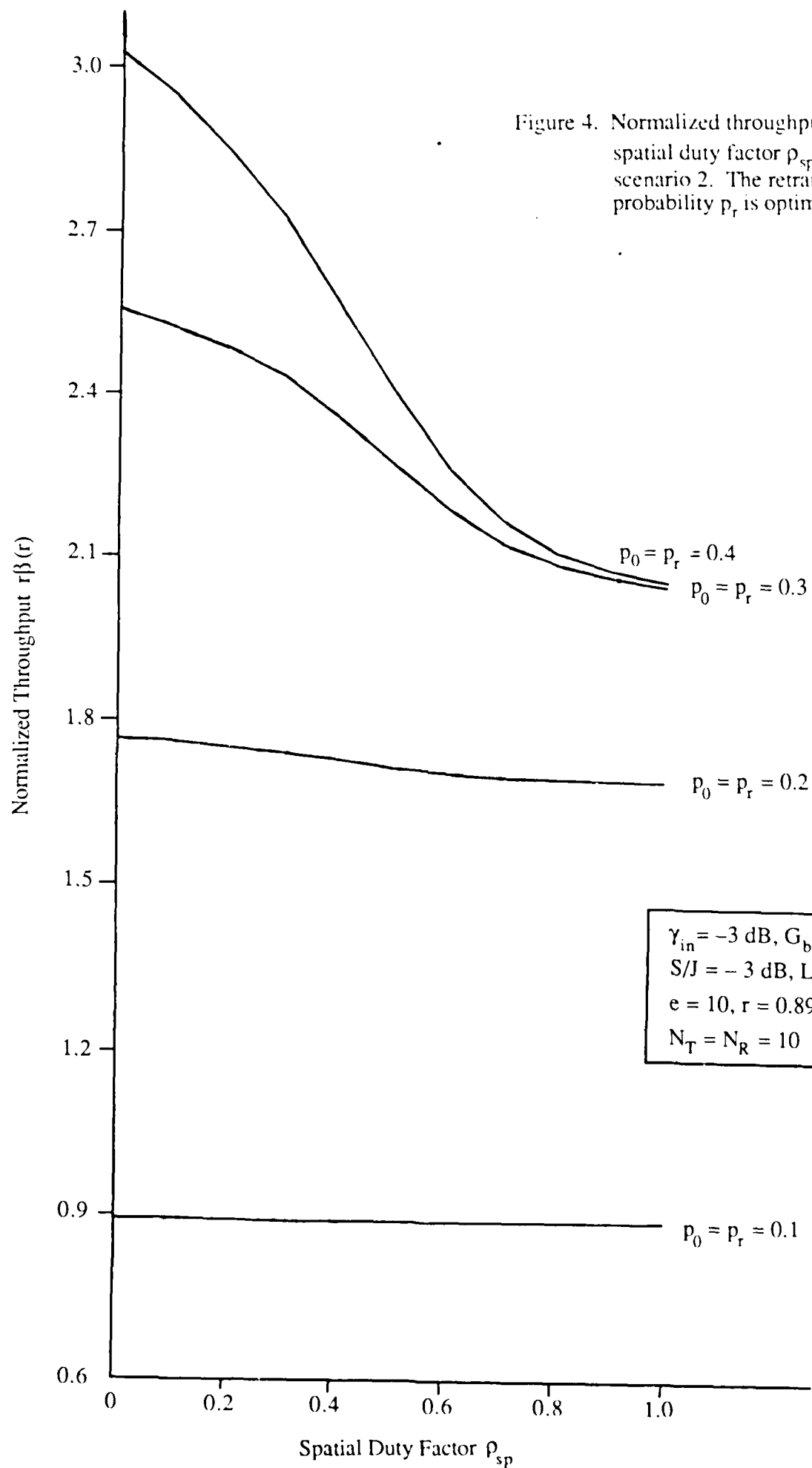
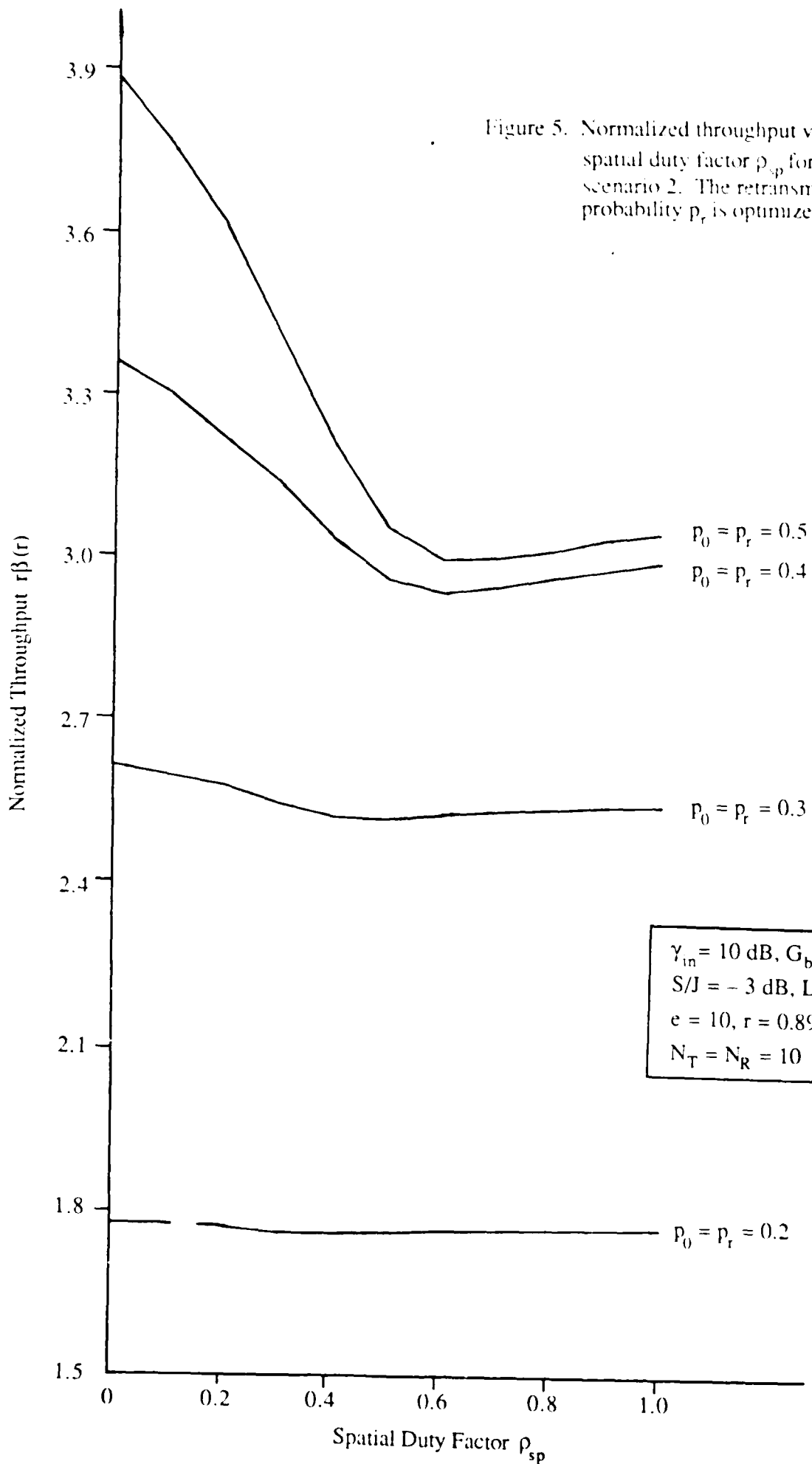




Figure 5. Normalized throughput versus the spatial duty factor  $\rho_{sp}$  for jamming scenario 2. The retransmission probability  $p_r$  is optimized.



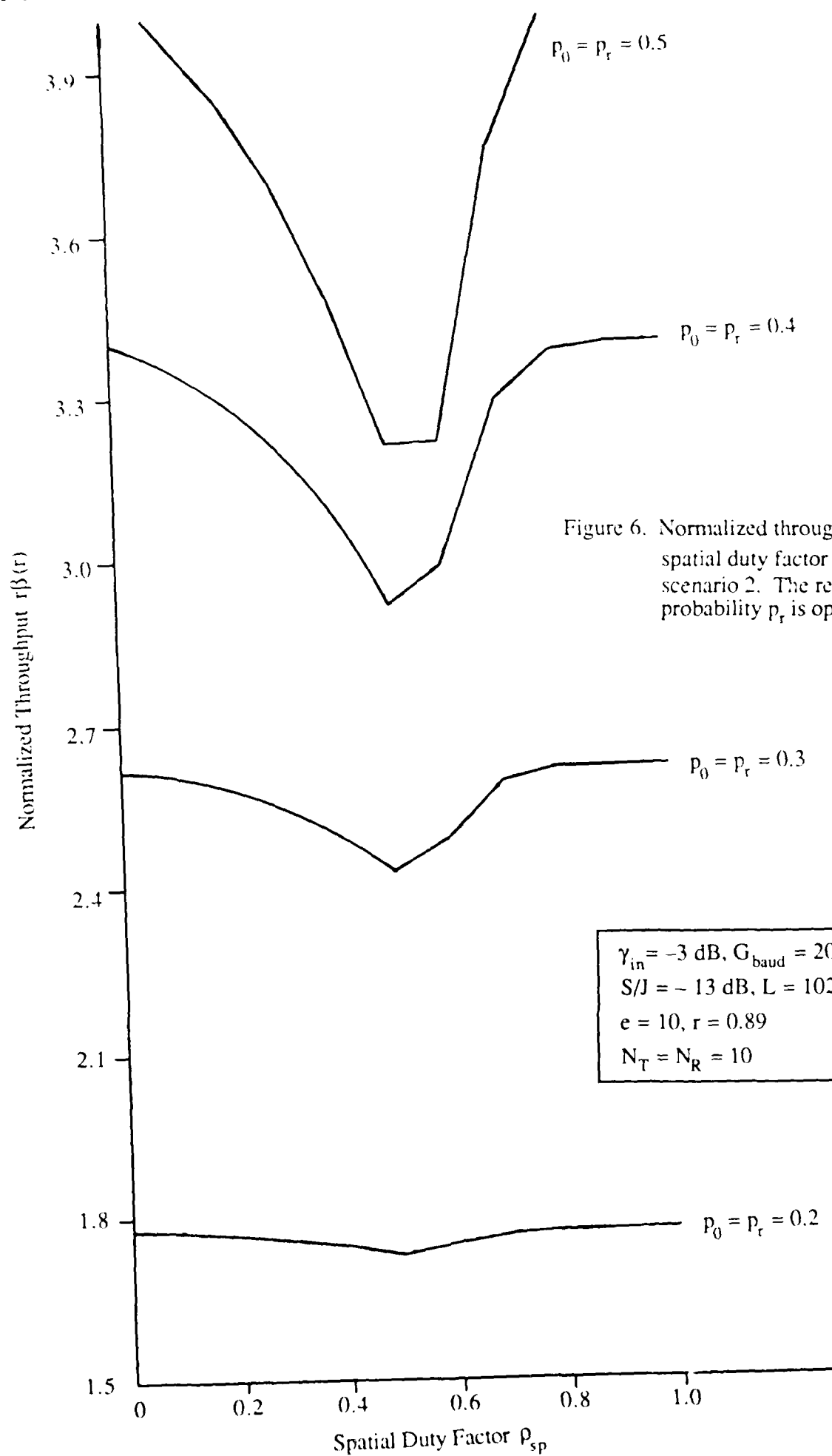
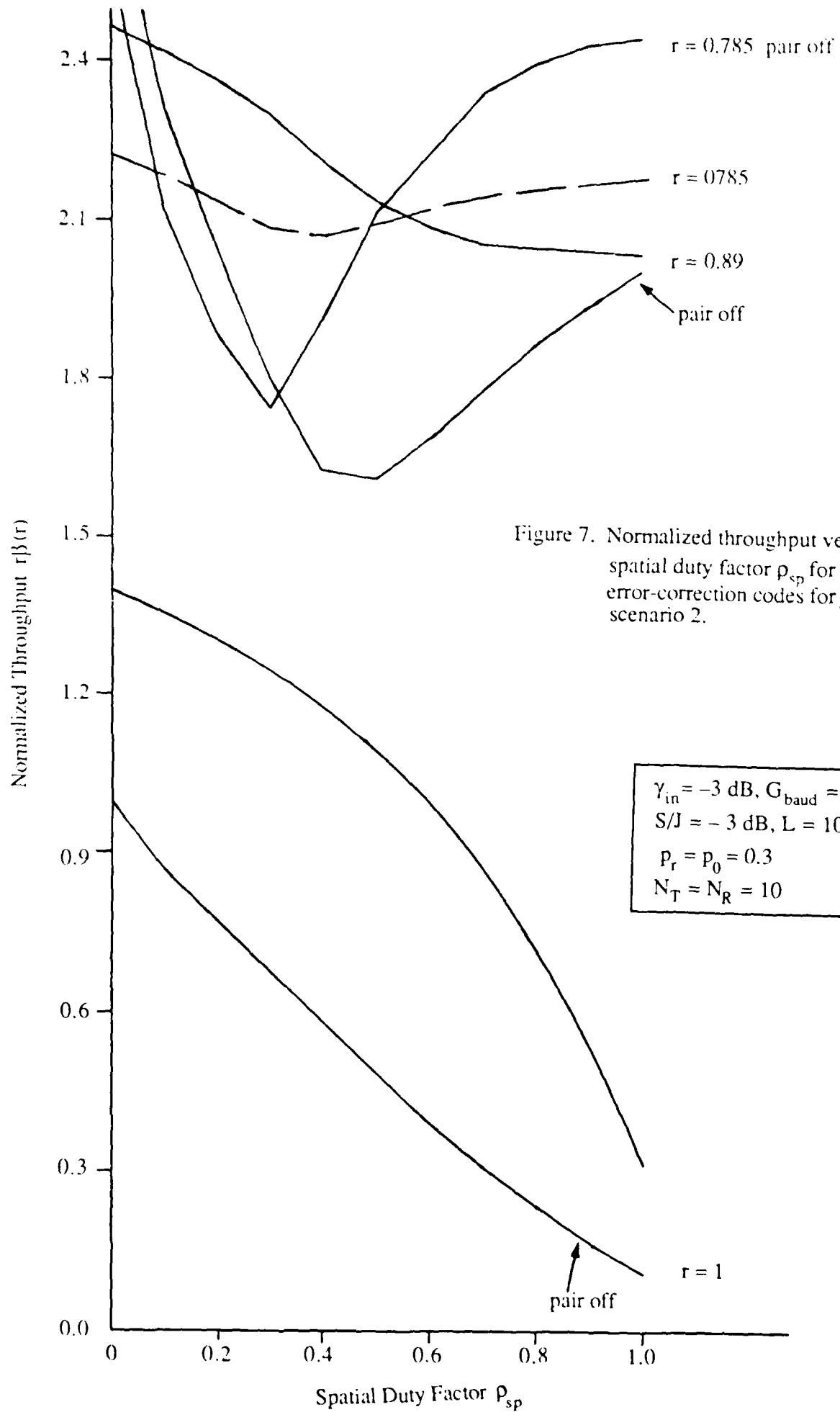
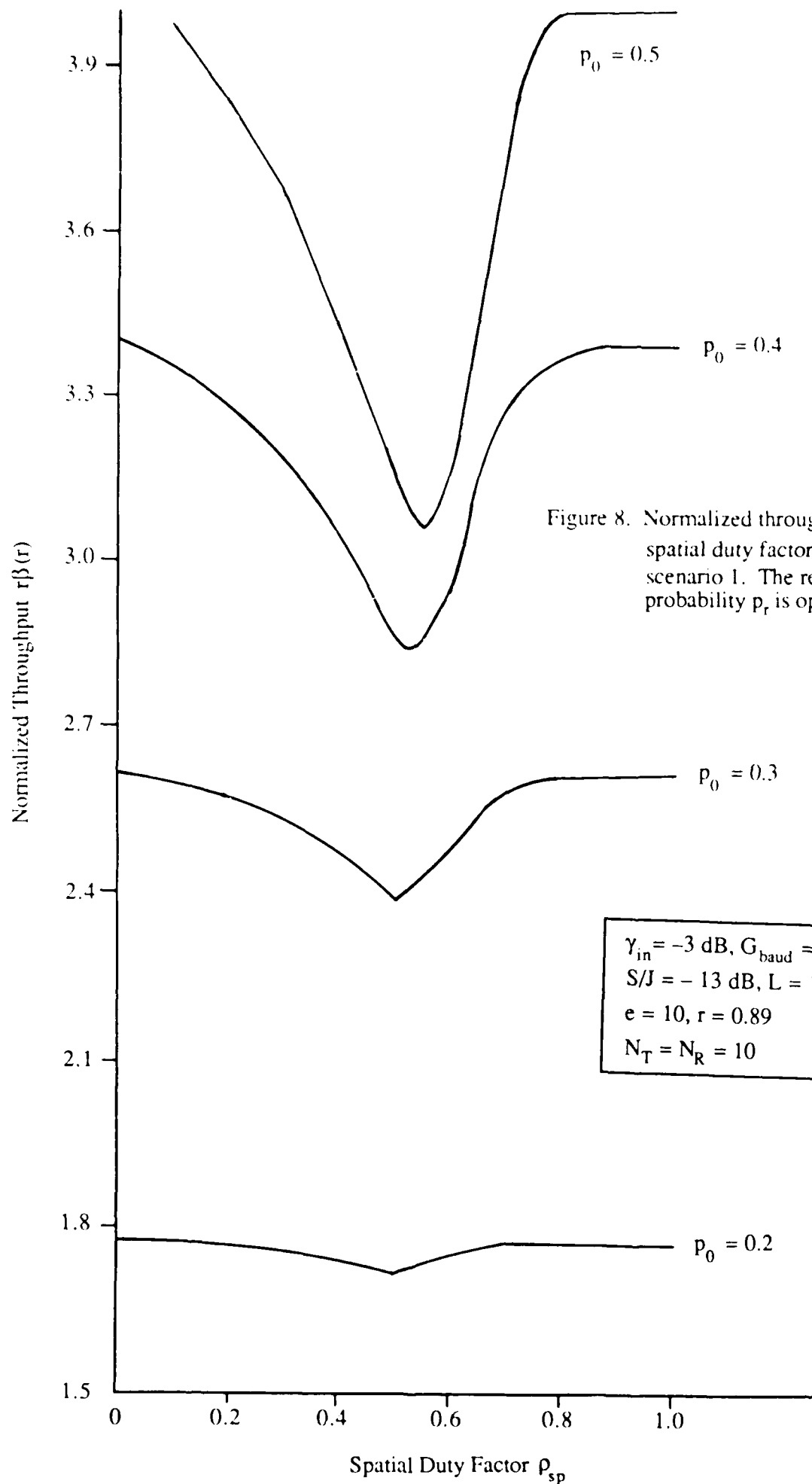


Figure 6. Normalized throughput versus the spatial duty factor  $\rho_{sp}$  for jamming scenario 2. The retransmission probability  $p_r$  is optimized.



In Figure 8, we show the normalized throughput versus the jammer spatial duty factor  $\rho_{sp}$  with  $p_0$  being the parameters for jamming scenario 1. The retransmission probability  $p_r$  is optimized in this figure. Compared with Figure 6, we see that the worst-case  $\rho_{sp}$  in scenario 1 is slightly more effective than that in scenario 2.



## Appendix A

As mentioned in Section 3b, for the jamming scenario 1, there are two ways to compute  $\overline{p_{sim}}$ , namely:

Method 1 Compute  $\overline{p_{sim}}$  using equation (22), where  $p_{sim,m_R^J}$  are computed by equation (24c) for each  $m_R^J$ .

Method 2 Compute  $\overline{p_{sim}}$  directly using recursion formula (24c) by substituting  $p_{sim,m_R^J}^{(k)}$  and  $p_A^{R_{k+1}}(m)$  for  $p_{sim}^{(k)}$  and  $\overline{p_A^R}$ , respectively, and identifying  $p_{sim,m_R^J}^{N_R} \equiv \overline{p_A^{(N_R)}}$ .

In this appendix, we prove that these two methods are equivalent.

For the  $1 \leq s \leq \min(m, k+1)$  case, we have

$$\begin{aligned} \overline{p_{sim}^{(k+1)}} &= \sum_{i=0}^{k+1} \binom{k+1}{i} \rho_{sp}^i (1 - \rho_{sp})^{k+1-i} p_{sim,i}^{(k+1)} \\ &= \sum_{i=0}^{k+1} \binom{k}{i} \rho_{sp}^i (1 - \rho_{sp})^{k-i} \left[ (1 - \rho_{sp}) p_{sim,i}^{(k+1)} + \rho_{sp} p_{sim,i+1}^{(k+1)} \right]. \quad (A-1) \end{aligned}$$

In the above equation, the first equality is due to equation (22), and the second equality is

obtained by applying the equality  $\binom{k+1}{i} = \binom{k}{i} + \binom{k}{i-1}$ . We then apply the recursion

formula (24c) to expand  $p_{sim,i+1}^{(k+1)}$  and  $p_{sim,i}^{(k+1)}$  in terms of  $p_{sim,i}^{(k)}$ . Note that, we use  $p_A^{R_{k+1}}(m, J^c)$  for  $p_{sim,i}^{(k+1)}$  and  $p_A^{R_{k+1}}(m, J)$  for  $p_{sim,i+1}^{(k+1)}$  in the expansion. Thus, we end up with

$$\begin{aligned}
\overline{p_{s,m}^{(k+1)}} &= \left[ \left( 1 - \overline{p_A^{R_{k+1}}} \right) + \frac{s}{m} \overline{p_A^{R_{k+1}}} \right] \sum_{i=0}^k \binom{k}{i} \rho_{sp}^i (1 - \rho_{sp})^{k-i} \overline{p_{s,m,i}^{(k)}} \\
&\quad + \frac{m-s+1}{m} \overline{p_A^{R_{k+1}}} \sum_{i=0}^k \binom{k}{i} \rho_{sp}^i (1 - \rho_{sp})^{k-i} \overline{p_{s,m,i}^{(k)}} \\
&= \left[ \left( 1 - \overline{p_A^{R_{k+1}}} \right) + \frac{s}{m} \overline{p_A^{R_{k+1}}} \right] \overline{p_{s,m}^{(k)}} \\
&\quad + \frac{m-s+1}{m} \overline{p_A^{R_{k+1}}} \overline{p_{s,m}^{(k)}}. \tag{A-2}
\end{aligned}$$

The last equality is obtained by applying equation (22) again and it is exactly Method 2.

Using the similar technique, it is easy to verify the  $s = 0$  case.

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